UDC: 621.375.826

PHONON COOLING CORRELATING DYNAMICS

Cârlig S.*
Institute of Applied Physics, Academy of Sciences of Moldova, Academiei str. 5, Chisinau, MD-2028, Republic of Moldova
*e-mail: carligsergiu@gmail.com

We investigated the unsteady-state cooling dynamics of vibrational quanta related to a nanomechanical oscillator coupled with a laser-pumped quantum dot in an optical resonator. Nanoresonator flexion modifies dot's energy levels and for a set of parameters absorption of laser photon and nanoresonator phonon is followed by photon emission in cavity mode. Such scheme of photon/phonon absorption/emission allows detection of nanoresonator cooling due to cavity photon emission.

Keywords: nanomechanical resonator, quantum dot, quantum cooling.

Am investigat dinamica nestaţionară a răcirii cuantelor de vibraţie ale unui oscilator nanomecanic cuplat cu un punct cuantic pompat din exterior cu radiaţie coerentă, plasat într-o cavitate optică. Flexiunile nanorezonatorului modifică nivelel energetice ale punctului cuantic şi pentru anumiţi parametri absorbţia unui foton laser, însoţită de absorbţia unui fonon al nanorezonatorului este urmată de emisia unui foton în cavitate. O astfel de schemă a emisiei/absorbţiei fotonilor/fononului permite detectarea răcirii datorită fotonului emis în cavitatea optică.

Cuvinte-cheie: rezonator nanomecanic, punct cuantic, răcire cuantică.

INTRODUCTION

The last years are marked by fulminant developing of nanotechnologies based particularly on mechanical structure like nano mechanical resonators (NMR) [1-3].

They are more attractive because of their capabilities to measure small and ultrasmall forces or masses with order 10^{-21} N or 10^{-21} g.

In addition, the "symbiosis"/coupling of NMR and optical resonators leads to improve the response to electromagnetic field with numerous applications such as single electron spin detection [1] or laser cooling [1, 4, 5].

Systems combining mechanical and optical subsystems give the necessary opportunities to quantum control mechanical object and quantum information processing.

The paper is outlined as follows: Section II presents the model Hamiltonian and approximation used to obtain master equation. In Section III the corresponding equations of motion are provided and obtaining results are analyzed, followed by a conclusion.

MODEL

We consider the situation presented in fig. 1, in which a semiconductor flexible beam is suspended in an optical cavity. Exteriorly laser pumped artificial two-level atomic structure is attached to the beam. The difference between ground and excited dot level is ω_0, laser and cavity frequencies are ω_L and ω_C, then nanoresonator one is ω.

Model Hamiltonian is given:

$$H = \hbar\omega_c a^{+}a + \hbar\omega b^{+}b + \hbar\omega_0 S_z + \hbar g\left(a^{+}S^{-} + aS^{+}\right) + \hbar\Omega\left(S^{+}e^{-i\omega_L t} + S^{-}e^{i\omega_L t}\right) + \hbar\lambda S_z\left(b^{+} + b\right) \quad (1)$$

where $S^{\pm}$ the first three terms describe the free energies of the artificial two-level system as well as of the optical and mechanical modes. The fourth and the fifth terms characterize the interaction of the quantum dot with the laser field and optical resonator mode, respectively. The last term takes into account the interaction of the vibrational degrees of freedom with the radiator [6].

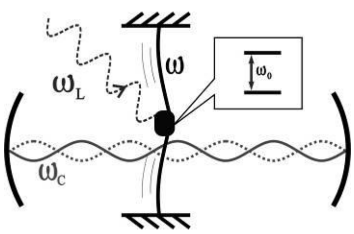

Fig. 1. Studied model consisting of semiconductor beam with attached quantum dot suspended in optical cavity

Correspondingly, g and λ denote the interaction strengths among the two-level emitter and the involved optical and mechanical modes, while Ω is the corresponding Rabi frequency due to external laser pumping. $S^{\pm}$, S_z are the qubit operators satisfying the standard commutation relations, while $\{a^{\dagger},a\}$ and $\{b^{\dagger},b\}$ are the generation and annihilation operators for photon and phonon subsystems, respectively, and obey the boson commutation relations [7].

In the Born-Markov approximation, then the whole system is rotated with laser frequency master equation is giving:

$$\frac{d}{dt}\rho(t)+\frac{i}{\hbar}[H,\rho]=-\gamma[S^{+},S^{-}\rho]-\gamma_c[S_z,S_z\rho]-\kappa_a[a^{\dagger},a\rho]-\kappa_b(1+\bar{n})[b^{\dagger},b\rho]-\kappa_b\bar{n}[b,b^{\dagger}\rho]+H.c., \qquad (2)$$

where γ and γ_c are the single-qubit spontaneous decay and dephasing rates, respectively, whereas $\kappa_a(\kappa_b)$ is the photon (phonon) resonator damping rate, and $\bar{n}$ is the mean-phonon number corresponding to temperature T and vibrational frequency ω. $\Delta = \omega_0 - \omega_L$ describes the detuning of the laser frequency from the two-level transition frequency, while $\Delta_1 = \omega_L - \omega_C$ accordingly is the detuning of the cavity frequency from the laser one.

RESULTS

Using the master equation (2), in the dressed states picture, the following equations of motion can be obtained for the mean photon and phonon numbers, respectively:

$$\frac{d}{dt}\langle a^{\dagger}a\rangle=\langle a^{\dagger}a\rangle(A_1-B_1+A_1^{*}-B_1^{*})+\langle a^{\dagger}b\rangle(C_2^{*}-D_2)+\langle b^{\dagger}a\rangle(C_2-D_2^{*})+A_1+A_1^{*},$$

$$\frac{d}{dt}\langle b^{\dagger}b\rangle=\langle b^{\dagger}b\rangle(A_2-B_2+A_2^{*}-B_2^{*})-\langle a^{\dagger}b\rangle(C_1^{*}-D_1)-\langle b^{\dagger}a\rangle(C_1-D_1^{*})+A_2+A_2^{*},$$

$$\frac{d}{dt}\langle a^{\dagger}b\rangle=\langle a^{\dagger}b\rangle\big(A_1^{*}-B_1+A_2-B_2^{*}-i(\Delta_1+\omega)\big)-\langle a^{\dagger}a\rangle(C_1-D_1^{*})+\langle b^{\dagger}b\rangle(C_2-D_2^{*})-C_1-D_2^{*},$$

$$\frac{d}{dt}\langle b^{\dagger}a\rangle=\langle b^{\dagger}a\rangle\big(A_1-B_1^{*}+A_2^{*}-B_2+i(\Delta_1+\omega)\big)-\langle a^{\dagger}a\rangle(C_1^{*}-D_1)+\langle b^{\dagger}b\rangle(C_2^{*}-D_2)-C_1^{*}-D_2. \qquad (3)$$

where:

$$A_1^{*}=\frac{1}{4}\frac{g^2\sin^2 2\theta}{\Gamma_{\text{‰}}-i\Delta_1}+\frac{g^2P_{-}\sin^4\theta}{\Gamma_{\perp}+i(2\Omega_R-\Delta_1)}+\frac{g^2P_{+}\cos^4\theta}{\Gamma_{\perp}-i(2\Omega_R+\Delta_1)},$$

$$A_2^{*}=\frac{1}{4}\Big(\frac{\lambda^2\cos^2 2\theta}{\Gamma_{\text{‰}}+i\omega}+\frac{\lambda^2P_{-}\sin^2 2\theta}{\Gamma_{\perp}+i(2\Omega_R+\omega)}+\frac{\lambda^2P_{+}\sin^2 2\theta}{\Gamma_{\perp}-i(2\Omega_R-\omega)}\Big)+\kappa_b\bar{n},$$

$$C_1^{*}=\frac{P_{+}}{2}\frac{g\lambda\sin 2\theta\cos^2\theta}{\Gamma_{\perp}-i(2\Omega_R+\Delta_1)}-\frac{P_{-}}{2}\frac{g\lambda\sin 2\theta\sin^2\theta}{\Gamma_{\perp}+i(2\Omega_R-\Delta_1)}-\frac{1}{4}\frac{g\lambda\sin 2\theta\cos 2\theta}{\Gamma_{\text{‰}}-i\Delta_1},$$

$$C_2^{*}=\frac{P_{-}}{2}\frac{g\lambda\sin 2\theta\cos^2\theta}{\Gamma_{\perp}+i(2\Omega_R-\omega)}-\frac{P_{+}}{2}\frac{g\lambda\sin 2\theta\sin^2\theta}{\Gamma_{\perp}-i(2\Omega_R+\omega)}-\frac{1}{4}\frac{g\lambda\sin 2\theta\cos 2\theta}{\Gamma_{\text{‰}}-i\omega}. \qquad (4)$$

with:

$$\Omega_R=\sqrt{\Omega^2+(\Delta/2)^2},$$

$$\Gamma=\gamma(1-\cos^2 2\theta)+\gamma_c\sin^2 2\theta,$$

$$\Gamma_{\perp}=4\gamma_0+\gamma_{+}+\gamma_{-},$$

$$\gamma_{+}=\gamma\cos^4\theta+\frac{\gamma_c}{4}\sin^2 2\theta,$$

$$\gamma_- = \gamma \sin^4\theta + \frac{\gamma_c}{4}\sin^2 2\theta,$$

$$\gamma_0 = \frac{1}{4}(\gamma \sin^2 2\theta + \gamma_c \cos^2 2\theta).$$

Dressed states population are:

$$P_+ = \frac{\gamma_-}{\gamma_+ + \gamma_-}, \text{ and } P_- = \frac{\gamma_+}{\gamma_+ + \gamma_-}.$$

In the system (3) coefficients B_i^* can be obtained from A_i^* by substitution $P_\mp \leftrightarrow P_\pm$, and adding κ_a to B_1^* or κ_b to B_2^*. D_i^* are obtained from C_i^* by changing $P_\mp \leftrightarrow P_\pm$, where $\{i \in 1,2\}$.

Mean photon and phonon numbers are drawn in fig. 2 for a set of parameters normed to γ (the single-qubit spontaneous decay).

After some oscillations a system can be described in the steady states approximation and the scheme of photon/phonon absorption/emission are giving in fig. 3, with steady states of the cavity mean-photon number and the vibrational NMR mean-phonon number, respectively in fig. 4.

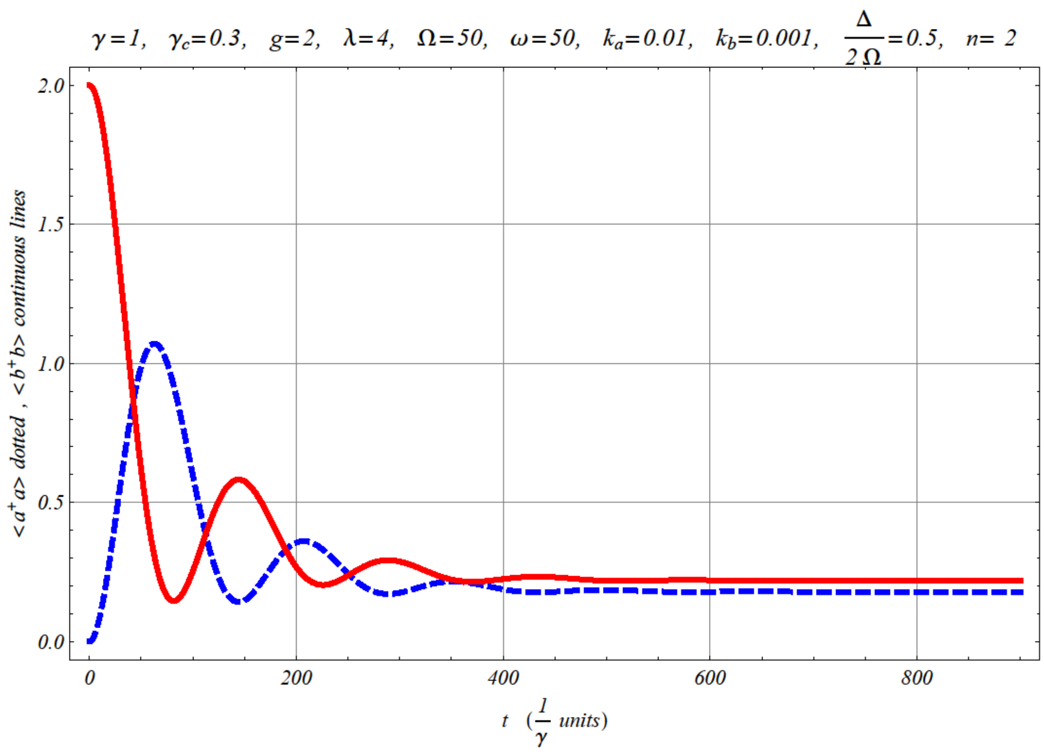

Fig. 2. Time dependence of the mean photon (dotted line) and phonon numbers (solid line)

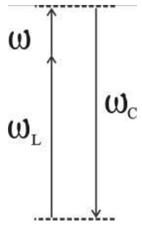

Fig. 3. Collective absorptions of a laser photon $\hbar\omega_L$ and a phonon $\hbar\omega$ are followed by cavity photon $\hbar\omega_C$ emission

The maximum photon detection corresponds to the NMR phonon minimum around $\Delta_1 + \omega \approx 0$.

Quantum cooling occurs for vibrational-mean-phonon numbers below those values imposed by the environmental incoherent reservoir. Furthermore, the quantum cooling is more efficient while increasing the temperature.

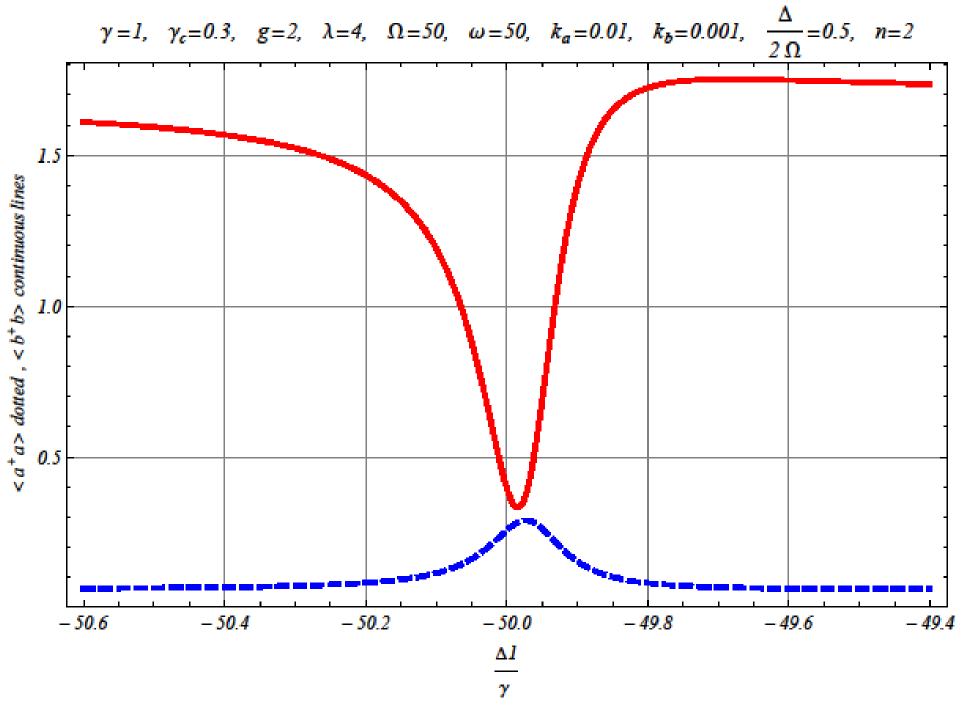

Fig. 4. The steady state mean-value of the photon $<a^+a>$, phonon $<b^+b>$ numbers as a function of Δ_1/γ Vibrational-mean-phonon number $<b^+b>$ is less than n (the mean phonon number imposed by the environmental incoherent reservoir)

CONCLUSION

We are obtained equations of motion for the mean photon and phonon number and showed their evolution. For steady states approximation can used a scheme to detect the vibrational phonon cooling of a nanomechanical oscillator. Correlating the vibrational degrees of freedom with those of a laser-pumped quantum dot when fixed on a nanomechanical beam while interacting with an optical cavity and when the quantum dot dynamics is faster than the corresponding ones of other involved subsystems, one needs to match the laser frequency such that both photon laser and NMR phonon absorption processes are accompanied by photon emission in the resonator mode. Therefore, detection of the cavity photons is followed by cooling of the nanomechanical oscillator.

BIBLIOGRAPHY

1. Markus Aspelmeyer, Tobias J. Kippenberg, Florian Marquardt. Cavity optomechanics, Rev. Mod. Phys. 86, p. 1391, 2014, arXiv:1303.0733.
2. Philipp Treutlein, Claudiu Genes, Klemens Hammerer, Martino Poggio, Peter Rabl. Hybrid Mechanical Systems, Quantum Science and Technology 2014, pp. 327-351, arXiv:1210.4151.
3. Гринберг Я.С., Пашкин Ю.А., Ильичёв Е.В. Наномеханические резонаторы, УФН, 182, 2012 pp. 407–436.
4. Aurélien Dantan, Bhagya Nair, Guido Pupillo, Claudiu Genes. Hybrid cavity mechanics with doped systems, Phys. Rev. A 90, 033820, 2014.
5. S. Cârlig and M. A. Macovei. Long-time correlated quantum dynamics of phonon cooling, Phys. Rev. A 90, 013817, 2014.
6. Laser Cooling of a Nanomechanical Resonator Mode to its Quantum Ground State, I. Wilson-Rae, P. Zoller, and A. Imamoglu, Phys. Rev. Lett. 92, 075507, 2004.
7. M. Kiffner, M. Macovei, J. Evers, C. H. Keitel. Vacuum-Induced Processes in Multilevel Atoms, Prog. Opt. 55, 85, 2010.

Prezentat la redacţie la 1 mai 2015