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QUANTUM DYNAMICS OF ACOUSTICAL PHONON STATISTICS

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We present a model of the quantum control via a laser light of the phonon statistics of an acoustical field and of the population inversion of a qubit. The phonon field is created in an acoustical multilayered nanocavity with a single-mode field being selected and it interacts with the thermal environment as well as with a qubit embedded in the cavity. The considered qubit is made of a quantum dot (QD). The confinement of the acoustical fields' quantum statistics is possible via driving the QD with an intense laser light, which for a proper detuning from the QD's transition frequency and for a well-chosen intensity may lead to sub-Poissonian distributed steady-state phonon fields. Furthermore, we show that for higher damping rates phonon assisted QD's population inversion occurs under the action of the created phonon fields with quantum statistics.

Keywords: quantum statistics, cuantum coherence, phonon laser, population inversion, quantum dots.

În acest articol este prezentat studiul controlului cuantic al statisticii fononilor a unui câmp acustic şi al inversiei de populaţie a unui qubit, folosind lumina unui laser. Câmpul fononic este creat într-o cavitate acustică multistrat, astfel fiind selectat un singur mod al câmpului ce interacţionează atât cu o baie termală cât şi cu un qubit încastrat în cavitate. Qubitul considerat este format dintr-un punct cuantic. Obţinerea şi controlul statisticii cuantice a câmpului fononic este posibilă prin utilizarea unui laser intens, care pentru o frecvenţă corect defazată de cea de tranziţie a punctului cuantic şi o intensitate bine aleasă poate duce la obţinerea câmpurilor fononice staţionare cu distribuţii sub-poissoniene. Mai mult, pentru rate de amortizare înalte, este obţinută inversia de populaţie a punctului cuantic sub acţiunea câmpurilor fononice, inversia fiind mai pronunţată pentru câmpuri cu statistică sub-poissoniană.

Cuvinte-cheie: statistică cuantică, coerenţă cuantică, laser fononic, inversie de populaţie, puncte cuantice.

INTRODUCTION TO THE MODEL

The generation of coherent phonons plays an important role in the quantum electrodynamics' (QEDs) research field, as phonon assisted processes reveal new quantum proprieties for QED setups, e.g., phonon assisted Mollow splitting [1], population inversion [2] or quantum statistics [3].

In the meantime, the study of the quantum dynamics of the acoustical fields, itself, leads to a new domain for QEDs, e.g., sub-Poissonian distributed phonon fields [4,5], phonon antibunching [6] and squeezing [7].

Remarkable results recently have been achieved in this domain for different experimental setups acting as an acoustical analog of the optical laser [8-10] and, furthermore, theoretical studies propose more new models [11-15] as well as new improvements [15,16].

In this paper, we describe a theoretical model of the generation of acoustical fields having sub-Poissonian distributions of quanta and we show that the created phonon fields may lead to the qubit's population inversion.

The qubit is made of a two level QD with a transition frequency ω_{qd} between its ground state $|g\rangle$ and its excited state $|e\rangle$ and it is embedded in an acoustical multilayered nanocavity [17] with a frequency ω_{ph} and a damping rate κ.

The single-mode phonon field is obtained by driving the QD by an intense laser with a frequency ω_L which interacts with the qubit in a semi-classical way with the Rabi oscillation Ω.

The QD interaction with the cavity's phonons is given by the coupling constant g.

The system's Hamiltonian consists of the free QD's and phonon field's terms and the QD-laser and QD-phonon interaction terms, given respectively as [18,19]:

$$H = \hbar\omega_{qd}S_z + \hbar\omega_{ph}b^\dagger b + \hbar\Omega\left(S^+e^{-i\omega_L t} + S^-e^{i\omega_L t}\right) + \hbar g S^+S^-(b^\dagger + b),$$

where the QD's operators are: $S^+ = |e\rangle\langle g|$, $S^- = |g\rangle\langle e|$, $S_z = 1/2\,(|e\rangle\langle e| - |g\rangle\langle g|)$ and the phonon creation and annihilation operators are $b^\dagger$and respectively b. The system's dynamics are solved by using the Lindblad form of the master equation [19]:

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}_{qd}\rho + \mathcal{L}_{ph}\rho,$$

where ρ is the density matrix operator and $\mathcal{L}_{qd}$, $\mathcal{L}_{ph}$ are the Liouville super-operators describing the damping phenomena, with $\mathcal{L}_{qd}$ corresponding to the QD's spontaneous emission and dephasing processes and $\mathcal{L}_{ph}$ describing the phonon damping by a thermal reservoir.

MODEL'S SOLUTIONS

The system dynamics, i.e., the previously mentioned master equation, are solved as follows. As a first step the system's Hamiltonian is transformed to a form which would give a solvable master equation. Then, the master equation is projected into QD's basis and, after some transformations in the phonon basis, leads to a system of infinite coupled equation. The transformations on the equations are made in order to obtain a system of equations that can be truncated. Once truncated it can be numerically solved and the parameters of interest can be deduced.

As a first step, we apply an unitary transformation to go in a frame rotating at the laser's frequency ω_L and then we use the dressed-state transformation defined by

$$|+\rangle = \sin\theta\,|\,g\,\rangle + \cos\theta\,|\,e\,\rangle,$$
$$|+\rangle = \sin\theta\,|\,g\,\rangle + \cos\theta\,|\,e\,\rangle,$$

where $\theta = 1/2\arctan(2\Omega/\Delta)$ and $\Delta = \omega_{qd} - \omega_L$. After one more unitary transformation $U(t) = e^{-iH't}$ corresponding to $H' = \hbar\bar{\Omega}R_z + \hbar\omega_{ph}b^\dagger b$, the system's Hamiltonian may be written as a sum of two terms corresponding to their rotation frequency, i.e., a slow rotating term and a fast rotating one by considering that $g \ll \{\omega_{ph}, \omega_{ph} + 2\bar{\Omega}\}$:

$$H = H_{slow} + H_{fast},$$
$$H_{slow} = -\hbar g\frac{\sin(2\theta)}{2}\{b^\dagger R^- e^{i(\omega_{ph}-2\bar{\Omega})t} + \text{H.c.}\},$$
$$H_{fast} = \hbar g(\sin^2\theta\,R_{--} + \cos^2\theta\,R_{++})\{b^\dagger e^{i\omega_{ph}t} + \text{H.c.}\} + \hbar g\frac{\sin(2\theta)}{2}\{b^\dagger R^+ e^{i(\omega_{ph}+2\bar{\Omega})t} + \text{H.c.}\},$$

where $R^+ = |+\rangle\langle -|$, $R^- = |-\rangle\langle +|, R_{++} = |+\rangle\langle +|, R_{--} = |-\rangle\langle -|$ are the new QD's operators in the dressed-state basis. Instead of applying a usual secular approximation by simply canceling the fast rotating terms, we consider their main contribution as follows:

$$H_{fast}^{eff} = -\frac{i}{\hbar}H_{fast}(t)\int dt' H_{fast}(t') = H_0 - \hbar\bar{\Delta}R_z + \hbar\beta b^\dagger b R_z,$$
$$\bar{\Delta} = \frac{g^2}{2}\left(\frac{\cos(2\theta)}{\omega_{ph}} - \frac{\sin^2(2\theta)}{4(\omega_{ph}+2\bar{\Omega})}\right), \qquad \beta = g^2\frac{\sin^2(2\theta)}{4(\omega_{ph}+2\bar{\Omega})}.$$

with H_0 being a constant term which can be dropped as it does not contributes to the system's dynamics. The new obtained Hamiltonian, i.e., $H = H_{slow} + H_{fast}^{eff}$, may be resumed to the secular approximation case if $\{\bar{\Delta}, \beta\} \approx 0$, which is not always the case. In our previous works we have already shown that the main contribution of the fast rotating terms is essential to describe the quantum proprieties of the phonon statistics [20]. A final unitary transformation is applied according to the slow rotating terms, so that the final system's Hamiltonian is:

$$H = \hbar(\omega_{ph} - 2\bar{\Omega})b^\dagger b - \hbar\bar{\Delta}R_z + \hbar\beta b^\dagger b R_z - \hbar g\frac{\sin(2\theta)}{2}(b^\dagger R^- + bR^+).$$

In order to introduce the final Hamiltonian in the master equation, similar transformation has to be applied to the damping terms. The phonon damping term rests unchanged and is given by:

$$\mathcal{L}_{ph}\rho = -\kappa(1+\bar{n})[b^{\dagger}, b\rho] - \kappa\bar{n}\,[b, b^{\dagger}\rho] + \mathrm{H.c.}\ ,$$

where the first term represents the damping and the last one the pumping processes of the interaction of the cavity's phonons with a thermal bath. The QD's damping term is given in the bare state basis as:

$$\mathcal{L}_{qd}\rho = -\gamma[S^{+}, S^{-}\rho] - \gamma_c[S_z, S_z\rho] + \mathrm{H.c.}\ ,$$

where γ and γ_c are, respectively, the QD's spontaneous emission and the dephasing rates. The dressed-state transformation leads to a new expression of $\mathcal{L}_{qd}$ after a secular approximation applied on the QD's counter-rotative terms:

$$\mathcal{L}_{qd}\rho = -\gamma_{+}[R^{+}, R^{-}\rho] - \gamma_{-}[R^{-}, R^{+}\rho] - \gamma_0[R_z, R_z\rho] + \mathrm{H.c.}\ ,$$

expressed by the QD's dressed-state decay rates:

$$\gamma_{+} = \gamma\cos^4\theta + \frac{1}{4}\gamma_c\sin^2(2\theta),$$

$$\gamma_{-} = \gamma\sin^4\theta + \frac{1}{4}\gamma_c\sin^2(2\theta),$$

and $\gamma_0 = \frac{1}{4}\left[\gamma\sin^2(2\theta) + \frac{1}{4}\gamma_c\cos^2(2\theta)\right]$.

After all the terms of the master equation are determined, one can solve the system dynamics by projecting the equation in the QD-phonon system's basis. The projection in the system's state basis in the steady-state regime leads to a set of infinite linear coupled equation. Once truncated by considering the asymptotic behavior of the phonon distribution, the system of equations can be numerically solved (*the complete method is given in* [20]).

RESULTS AND DISCUSSIONS

The focus of this study is made on the statistics of the created steady-state phonon fields in the acoustical nanocavity and, furthermore, we investigate how the created phonon field influences the quantum dot state in the steady-state regime. The entire process is controlled by the laser's parameters and different nanocavities with different damping rates are used. In order to characterize the phonon field, we investigate the behavior of the second-order correlation function $g^{(2)}(0)$, which equals unity for coherent phonons, goes below unity for quantum fields with sub-Poissonian distribution and goes above unity for classical fields with $g^{(2)}(0) = 2$ for thermal fields. The QD's behavior is described by the population inversion W term, which has negative values when the QD is more likely to be in the ground state and positive values for the QD more probably to be found in the excited state.

Two different damping regimes are observed for the studied model, corresponding to high and low damping rates. The first case shown in fig. 1(b), corresponding to high damping rates of the order of $\kappa/\gamma \approx 10$, is described, for a well-chosen detuning, by weak phonon fields with a prominent sub-Poissonian distribution. Under the phonon field's action, the QD's population is inverted in the region where the field is more intense and reaches the maximum level in the region of sub-Poissonian fields. Thus, the information about the QD's state can be obtained by monitoring the phonon fields' statistics and vice versa.

In the second case shown in fig. 1(a), for low damping rates of the order of $\kappa/\gamma \approx 10^{-3}$, the acoustical field's statistics are modified to a less prominent sub-Poissonian behavior but a higher mean phonon number in the cavity. The population inversion is always negative, so that this regime is of no interest for monitoring the QD's state. However, for the studies of the phonon fields' statistics only, we show that quite strong fields with quantum statistics may be obtained in an enough realistic case.

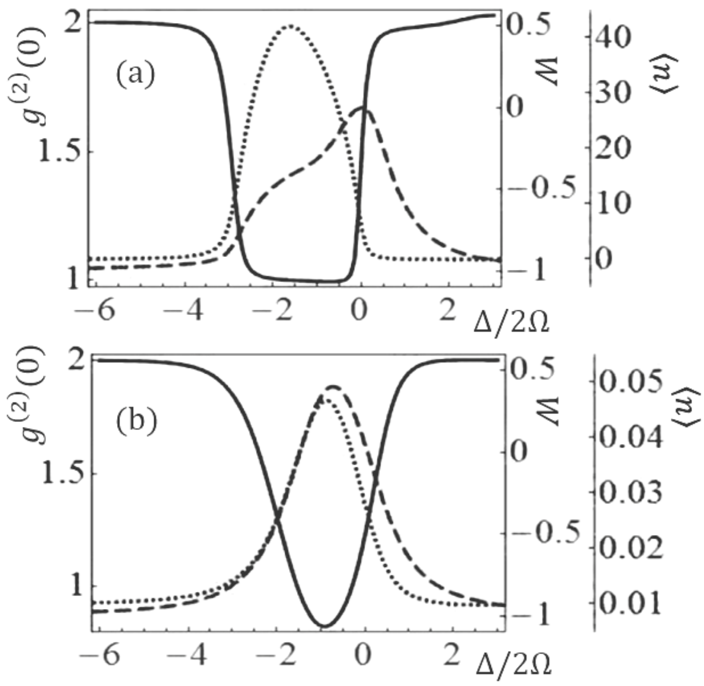

Fig. 1. The second-order correlation function $g^{(2)}(0)$ (continuous curve), the mean phonon number $\langle n \rangle$ (dotted curve) and the QD's population inversion W (dashed curve) as functions of the laser's detuning $\Delta/2\Omega$.
(a) For a low damping regime with $\kappa/\gamma = 5 \times 10^{-3}$ and (b) for a high damping regime with $\kappa/\gamma = 15$. The vertical axis are representing, from left to right: $g^{(2)}(0)$, W and $\langle n \rangle$. The other model's parameters are: $\bar{n} = 0.01$, $2\Omega/\gamma = 25$, $\gamma_c/\gamma = 0.1$, $\omega_{ph}/\gamma = 35$, $g/\gamma = 25$

As about the control parameters, i.e., the drive laser's parameters, beyond a well-chosen detuning, a moderate laser's intensity is required, corresponding to $\Omega/\Delta \approx 10 - 20$ in our case. For higher intensities, the phonon statistics does not manifest quantum proprieties and the population inversion's values decrease.

CONCLUSIONS

The proposed model showed that phonon fields showing pure quantum features as sub-Poissonian distributed quanta might be obtained for different damping regimes. First regime is related to low damping rates where quite intense sub-Poissonian distributed phonon fields are obtained. The second regime, related to high damping rates, corresponds to the case where the QD's population is inverted by the created phonon fields. Moreover, we show that for this regime the maximum of the population inversion is located in the region were the phonons have are more prominent sub-Poissonian distributions and a more intense field.

BIBLIOGRAPHY

1. Y.-J. Wei, Y He, Y.-M. He, Ch.-Y. Lu, J.-W. Pan, Ch. Schneider, M. Kamp, S. Höfling, D. P. S. McCutcheon, A. Nazir, Phys. Rev. Lett. 113, 097401 (2014).
2. S. Das, M. A. Macovei, Phys. Rev. B 88, 125306 (2013).
3. S. Carlig, M. A. Macovei, Phys. Rev. A 90, 013817 (2014).
4. H. Wang, Z. Wang, J. Zhang, S. K. Ozdemir, L. Yang, Yu-xi Liu, Phys. Rev. A 90, 053814 (2014).
5. H. Wu, G. Heinrich, F. Marquardt, New J. Phys. 15, 123022 (2013).
6. R. Okuyama, M. Eto, T. Brandes, New J. Phys. 15, 083032 (2013).
7. A. Kronwald, F. Marquardt, A. A. Clerk, Phys. Rev. A 88, 063833 (2013).
8. I. Mahboob, K. Nishiguchi, A. Fujiwara, H. Yamaguchi, Phys. Rev. Lett. 110, 127202 (2013).
9. I. S. Grudinin, H. Lee, O. Painter, K. J. Vahala, Phys. Rev. Lett. 104, 083901 (2010).

10. K. Vahala, M. Hermann, S. Knünz, V. Batteiger, G. Saatho, T. W. Hansch, Th. Udem, Nat. Phys. 5, 682 (2009).
11. H. Wu, G. Heinrich, F. Marquardt, New J. Phys. 15, 123022 (2013).
12. J. Kabuss, A. Carmele, T. Brandes, A. Knorr, Phys. Rev. Lett. 109, 054301 (2012).
13. J. Kabuss, A. Carmele, A. Knorr, Phys. Rev. B 88, 064305 (2013).
14. R. Okuyama, M. Eto, T. Brandes, New J. Phys. 15, 083032 (2013).
15. H. Wang, Z. Wang, J. Zhang, S. K. Ozdemir, L. Yang, Yu-xi Liu, Phys. Rev. A 90, 053814 (2014).
16. H. Jing, S. K. Ozdemir, Xin-You Lu, J. Zhang, L. Yang, F. Nori, Phys. Rev. Lett. 113, 053604 (2014).
17. O. O. Soykal, R. Ruskov, Ch. Tahan, Phys. Rev. Lett.107, 235502 (2011).
18. M. Kira, S. W. Koch, Semiconductor Quantum Optics, Cambridge University Press, Cambridge, (2012).
19. M. Scully, M. S. Zubairy, Quantum Optics (Cambridge University Press, Cambridge, 1997).
20. V. Ceban, M. A. Macovei, arXiv:1502.01198 [quant-ph] (2014).

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