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THE EXPERIMENTAL STUDY OF REAL AND IDEAL HARMONIC OSCILLATORS

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The following research was dedicated to studying and experimenting with harmonic oscillators of all kinds. The used equipment was a portable computer, the "PASCO Spark", a force sensor, 3 different springs and weights. The portable computer is responsible for storing the data received from the force sensor, and constructing a "Force-Time" graph. The force sensor has a range of ±50 N, an accuracy of 0.1 N and a recording speed of up to 1000 Hz. It is also equipped with a reset button to automatically set it to 0 N. The springs used in the experiment differed in damping. The first spring had a very low damping coefficient, so after the weight was released they continued oscillating for a very long time, fading away only very slightly. The second spring had a medium damping coefficient, so the time it took for the oscillations to fade was much shorter, then with the first spring. The third spring however had a very high damping coefficient, so the oscillations only continued for about 6 seconds. The experiment itself was performed by hanging a weight on the spring, and starting the data recording as soon as the weight is dropped. In all 3 experiments the data was recorded at a frequency of 100 Hz. Afterwards, the computer built "Force-Time" graphs for every experiment. This allowed us to visualize the difference between the 3 springs. The computer used in the experiment has a very useful ability of generating functions for graphs. This can be done in order to study ideal harmonic oscillations. The function generated from the graph continues forever, and can be used for further research in the field of beats, which are automatically calculated and simulated by the computer. Thanks to the precise data recording equipment human errors can be minimized, which leads to very little inaccuracies in calculations.

Keywords: harmonic oscillator, sensor, accuracy, simulation.

Cercetări de faţă sunt dedicate studierii şi experimentării cu oscilatoare armonice de toate tipuri. Echipamentul utilizat era calculator portativ „PASCO Spark", senzor de forţă, 3 arcuri diferite şi greutăţi. Calculator portativ este responsabil de stocarea datelor primite de la senzorul de forţă şi de construirea dependenţei „Forţă-Timp". Senzorul de forţă are diapazonul ±50 N, precizie de 0,1 N şi viteza de înregistrare de pînă la 1000 Hz. Acesta este, de asemenea, echipat cu un buton de resetare pentru a-l ajusta automat la 0 N. Arcurile folosite în experiment diferă după coeficientul de elasticitate. Primul arc avea coeficientul de elasticitate foarte mic, astfel încît după ce arcul era încărcat cu greutate ei continua să oscileze pentru un interval de timp îndelungat, dispărea doar foarte încet. Al doilea arc avea coeficientul de elasticitate mediu, astfel timpul de amortizare era mult mai scurt, decît la primul arc. Însă al treilea arc avea coefficient de elasticitate foarte mare, întrucît oscilaţiile au continuat numai aproximativ 6 s. Însăşi experimentul a fost efectuat prin atîrnarea greutăţii de arcul, iar înregistrarea datelor a început îndată ce greutatea era eliberată. În toate cele 3 experimente înregistrarea datelor a fost efectuată la frecvenţa de 100 Hz. Pe urmă calculatorul construia dependenţa „Forţă-Timp" pentru fiecare experiment. Aceasta ne-a permis să vizualizăm diferenţa dintre cele trei arcuri. Calculatorul utilizat în experiment are o capacitate foarte utilă de a genera funcţii pentru grafice. Acest lucru poate fi făcut în scopul de a studia oscilaţii armonice ideale. Funcţia generata din grafic este incontinuă şi poate fi utilizată pentru continuarea cercetărilor în domeniul bătăilor, care sunt calculate automat şi simulate la calculator. Datorită echipamentului precis de înregistrare a datelor erorile umane pot fi minimizate, ceea ce duce la erori foarte mici în calcule.

Cuvinte-cheie: oscilator armonic, senzor, precizie, simulare.

INTRODUCTION

The following article studies real and ideal harmonic oscillators. With the help of the experimental equipment I was able to record accurate data, do some research and visualise the difference between ideal and real harmonic oscillators.

To understand the concept of harmonic oscillators, let's see their official definition. A harmonic oscillator is a system that, when displaced from its equilibrium position, experiences a restoring force, *F*, proportional to the displacement, *x*. This can be put into the following equation:

$$\vec{F} = -k\vec{x} \tag{1}$$

This formula is derived from Newton's second law of motion.

IDEAL HARMONIC OSCILLATORS

For simple (or ideal) harmonic oscillators the damping force is taken as 0, so the system only depends on the mass of the oscillating object and the distance from the point where x = 0. With this in mind Newton's second law of motion takes the following form:

$$F = ma = m\frac{d^2x}{dt^2} = m\ddot{x} = -kx \quad (2)$$

By resolving the equation we are able to derive a function describing the motion of the oscillator:

$$x(t) = A\cos(\omega t + \varphi) \quad (3)$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

The oscillator's motion is repeating continuously with amplitude A. The period of one oscillation can also be found from the following equation:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad (4)$$

Or, if the frequency of the oscillations is known, the time would simply be equal to the inverse of the frequency:

$$T = \frac{1}{f} \quad (5)$$

The position of the oscillator also depends on the phase, φ, which determines the starting point on the sine wave. Ideal harmonic oscillations continue forever with the same amplitude due to the fact that damping is not present.

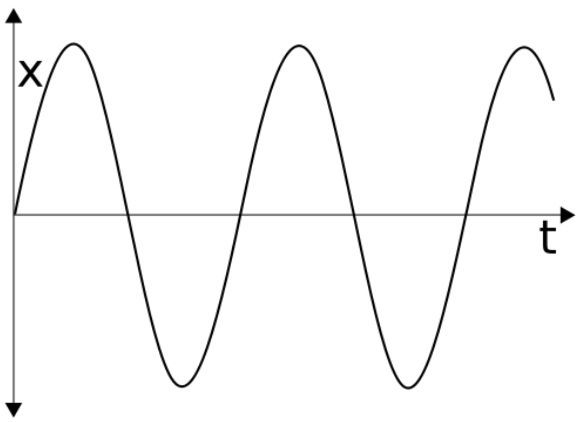

Fig. 1. Asimple x(t) graph describing the motion of an ideal harmonic oscillator

REAL HARMONIC OSCILLATORS

But in reality it is not possible to find ideal harmonic oscillators, since there will always be a friction force. Due to this force the velocity will decrease in proportion to the friction force acting on it. This relation can be shown in the following equation:

$$F_f = -cv, \quad (6)$$

where c is the viscous damping coefficient.

So the balanced equation with all the forces included will look like this:

$$F = F_{ext} - kx - cv, \quad (7)$$

where F_{ext} is the external force applied on the system.

Since friction can't be reduced to 0, in reality only such oscillators can exist, which gives them their name, "real oscillators".

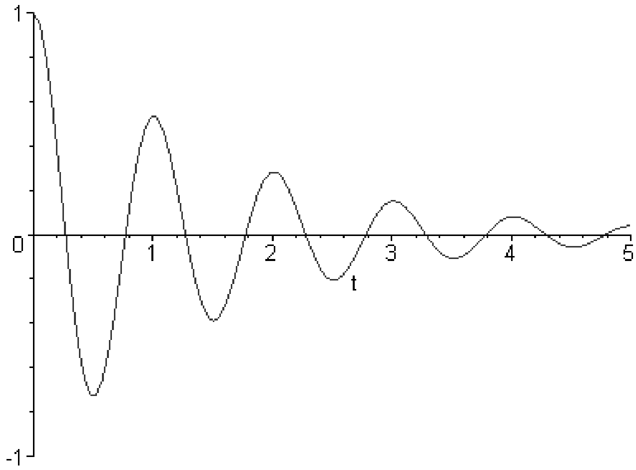

Fig. 2. An underdamped real harmonic oscillator gradually slowing down with time

The best way to illustrate a real harmonic oscillator would be a simple setup of a weight, attached to a fixed spring, oscillating up and down.

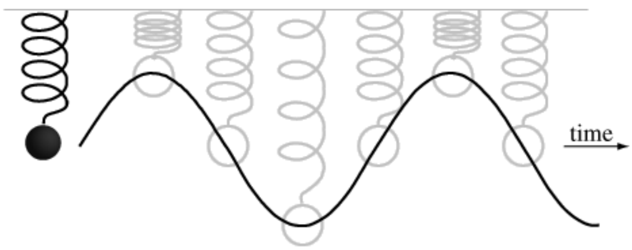

Fig. 3. An illustration of a harmonic oscillator in motion

EQUIPMENT

To do experiments with harmonic oscillators we used two different springs and weights, along with a force sensor and a computer. The spring was attached on the force sensor, and the weight was hung on the spring. The computer used in the experiment was the “Pasco SPARK science learning system”, and the force sensor was the “Pasco PASPORT”.

Pasco SPARK Science Learning System:

SPARK Science Learning System is an all-in-one mobile device that seamlessly integrates the power of probeware with inquiry-based content and assessment. With its large, full-color display, finger-touch navigation and completely intuitive data collection and analysis capabilities, SPARK completely redefines the concept of easy-to-use, so the focus remains on the learning of science.

- Full color, large screen for easy viewing;
- Finger-touch navigation – no stylus, ever!
- Simple two-button design;
- Temperature and voltage sensors included;
- Able to connect with a variety of sensors to work with all fields of science;
- More than 60 free guided inquiry SPARK labs pre-installed.

Fig. 4. The Pasco SPARK science learning system

Pasco PASPORT force sensor:

The study of force is critical to many science explorations. An accurate and rugged sensor will ensure your students get the most out of their force experiments. An overload stop in the force beam and a polycarbonate plastic case protect the unit from student abuse.

- Easy to zero - just press the button and the zero setting is stored digitally. No confusing manipulations of data are necessary.
- Measures forces in the direction of the sensor - side forces are minimized.
- Able to connect with the Pasco SPARK Science Learning System.
- Range: ±50 N.
- Accuracy: 1 %.
- Resolution: 0.03 N.
- Maximum data collecting frequency: 1000 Hz.

Fig. 5. The Pasco PASPORT force sensor

EXPERIMENTAL SETUP

The experimental setup is fairly simple and does not require a lot of time to prepare. On an elevated spot, the force sensor is hung. A spring with a weight on it is placed on the hook of the force sensor. The sensor is then connected to the virtual laboratory.

The computer is programmed to generate a Force-Time graph as soon as the measurements start. Before performing the experiment the force sensor is set to zero to make it easier to perform calculations and observations. As soon as everything is ready, the weight is elevated at a height and released to bounce on the spring.

The recording process is started at the same time. After a few seconds of bouncing, the graph can already be seen and analysed.

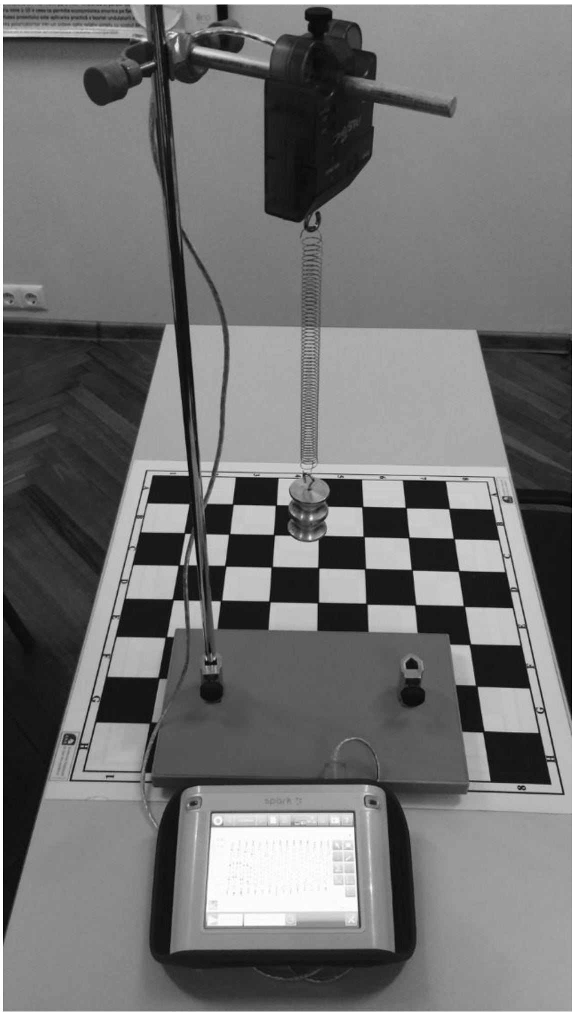

Fig. 6. The experimental setup in action

EXPERIMENT

After preparing the stand with all the equipment put in place and turned on, we started with a spring with a very low damping coefficient and a small weight. We hung the weight on the spring and reset the force sensor using the reset button on it, so that the spring hanging with the weight on it would be 0 N. Then we carefully lifted the weight to a height of a few centimetres, and released it.

At the same time on the computer we began the data collection at a rate of 100 Hz. The graph was set to be a Force-Time graph. After leaving the system oscillating for a few seconds the graph was ready.

The graph proves that the oscillator is damped only very slightly, since the force amplitude is practically staying the same. Of course, if the experiment was long enough, a decrease in motion would be clearly visible. In 7 seconds 12 oscillations occurred.

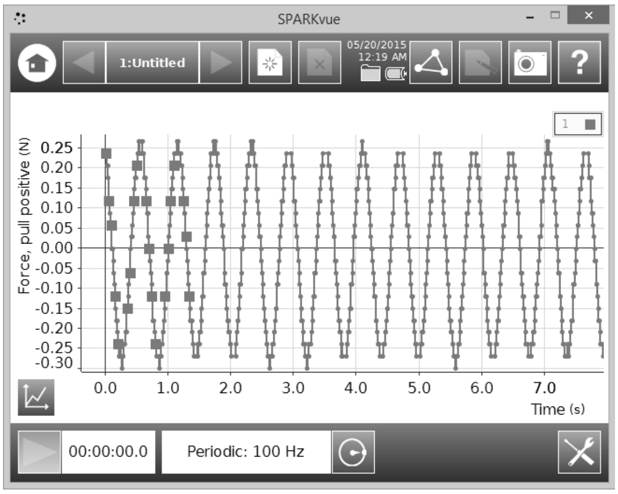

Fig. 7. The resultant graph of an almost perfect real harmonic oscillator

Using this data we can calculate $T = \frac{7}{12} \approx 0.58s$.

From such a graph it is very easy to calculate the time of one oscillation. Thanks to the precise measurements made by the laboratory we can clearly see the amount of oscillations and time.

To compare the resultant graph to ideal harmonic oscillations a function has to be created from the graph. Luckily, the virtual laboratory has such a function built into it. Since we already know that harmonic oscillators behave in a sinusoidal pattern, a sin fit function would fit the graph.

The function is generated using average values of the graph, and the overlapping of the function and graph show how much the real oscillator is different from an ideal one.

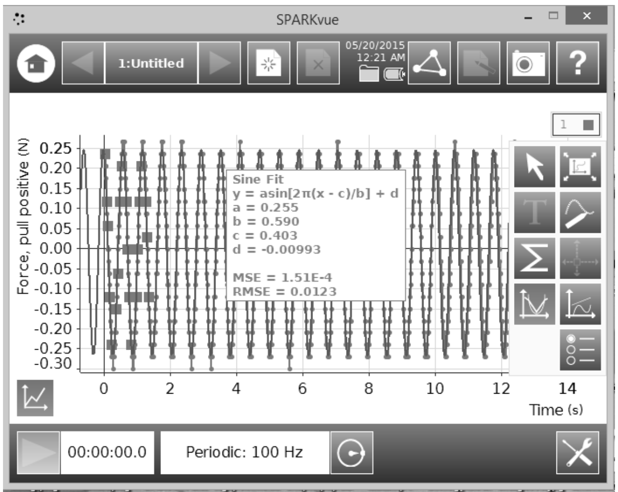

Fig. 8. Graph with a Sine Fit function overlapping it

This image again proves that the oscillations were practically perfect, since the function almost perfectly overlaps the graph.

After getting this result, we decided to redo the experiment with a different, fairly damped spring and larger weight. So we removed the previously installed spring and weight and installed the new ones. Again, we reset the force sensor to 0 N, saved the previous graph and opened a new, fresh one.

Already a few seconds after we released the weight we could clearly see that the damping was pretty strong and the oscillations were gradually slowing down. In a very short time the oscillations faded almost completely.

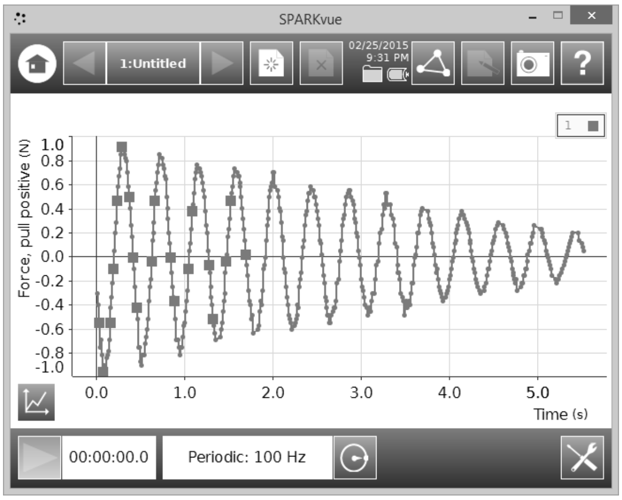

Fig. 9. The resultant graph of an underdamped mechanical oscillator

In this graph it's clearly visible how the oscillator was gradually slowing down throughout the experiment due to the damping.

After generating a Sine Fit function for the second graph it was clearly visible how much the oscillations were being slowed down by the damping of the system.

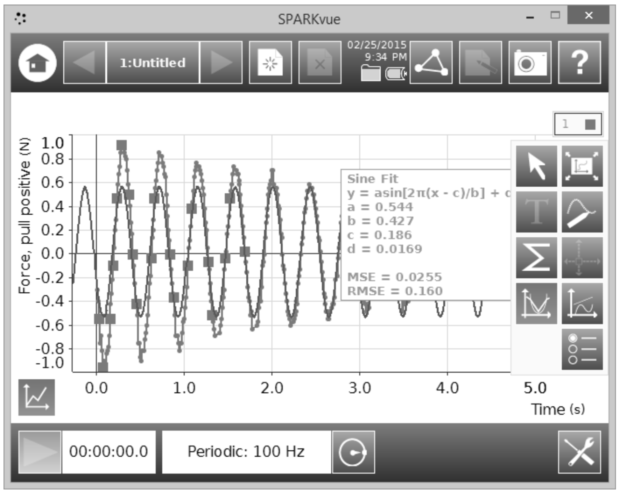

Fig. 10. Sine Fit function generated from average values of the graph

Since the average values were taken to generate an ideal harmonic oscillator function, its initial values differ from the graph. But nevertheless, observing this clearly shows the effect of damping on a harmonic oscillator.

CONCLUSIONS

Harmonic oscillations are a very interesting field of study and research. It's definitely fun to play around with them while performing calculations and measurements. In this experiment we observed real harmonic oscillators and compared them to their ideal counterpart.

Thanks to the great equipment it was very easy to explore this subject of study and to do precise calculations.

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